3D Pattern Formation in Turing Systems

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Project homepage: http://www.lce.hut.fi/research/polymer

1. Introduction

In the 1950s Alan Turing proposed a simple reaction−diffusion system to account for morphogenesis [1]. The general form of a Turing system for modeling the time evolution of morphogen concentrations is given by two coupled partial differential equations

$$u_t = D_u \Delta u + f(u,v)$$
$$v_t = D_v \Delta v + g(u,v),$$

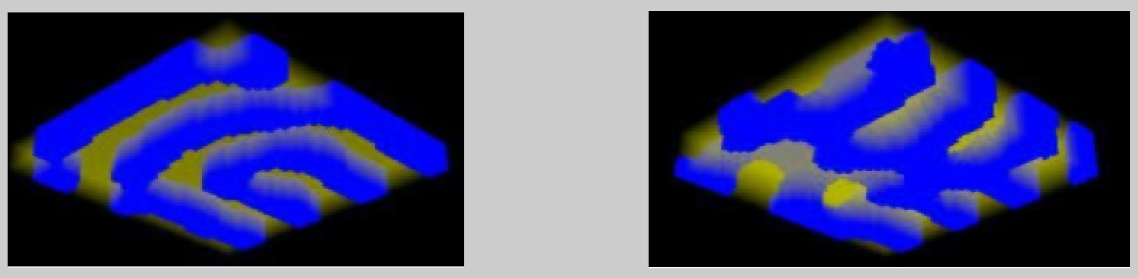
where *u(x,t)* and *v(x,t)* denote the concentrations of the chemicals. This pair of equations simply implies that the time variation of concentrations (left sides) depends on both the diffusion (right sides, first terms) and the reactions (right sides, second terms) of the chemicals. The functions *f* and *g* describing the reactions are typically nonlinear and the general Turing model we have used reads as

$$u_t = \delta D \Delta u + \alpha u (1 - r_1 v^2) + v (1 - r_2 u)$$
$$v_t = \delta \Delta v + v (\beta + \alpha r_1 u v) + u (\gamma + r_2 v)$$

Turing's ground−breaking idea was that random fluctuations around steady−state may cause bifurcation of the state due to diffusion. This is followed by the time evolution of the morphogen concentrations due to diffusion−driven instability. Morphogenesis results in patterns in 2D or structures in 3D. The most important characteristic of the Turing patterns is that whether the morphogenesis results in wide stripes or small spots is determined solely by the diffusion and reaction rates, i.e., parameter values of the system. The result of morphogenesis is essentially unaffected by initial conditions, dimensions of the system, boundary conditions and even a reasonable amount of random noise introduced to the system.

3. Future work

At the moment we are studying the effect of dimensionality, i.e., the transition between 2D and 3D.



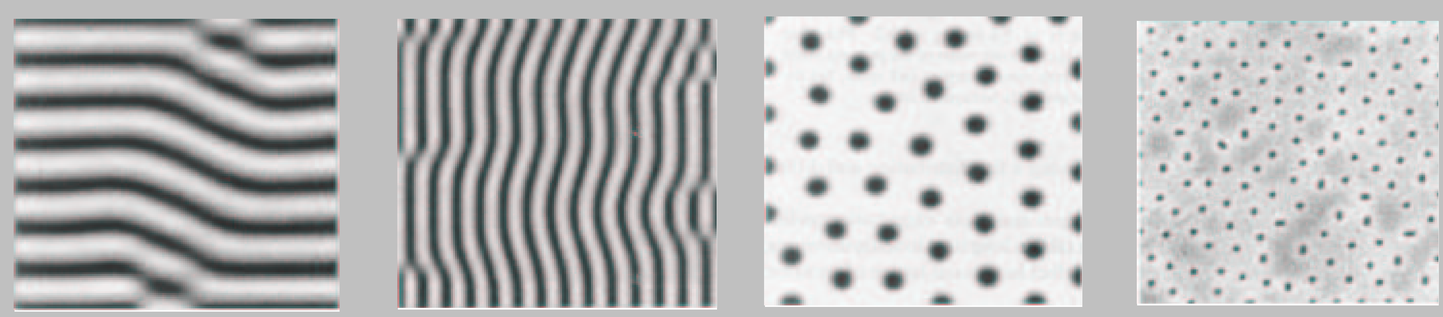
50x50x4 50x50x8

−The effect of random noise is of interest, since morphogenesis in the biological sense, has to sustain noise.
−Based on preliminary results the spherical phase manifests itself clearly even in the presence of very heavy noise.
−Time dependent oscillatory phases can be observed in Turing systems. In these cases the pattern formation process becomes even more complicated.
−Modeling of biological growth with a Turing model.
−See the project home page for videos and updates!

2. Research interests

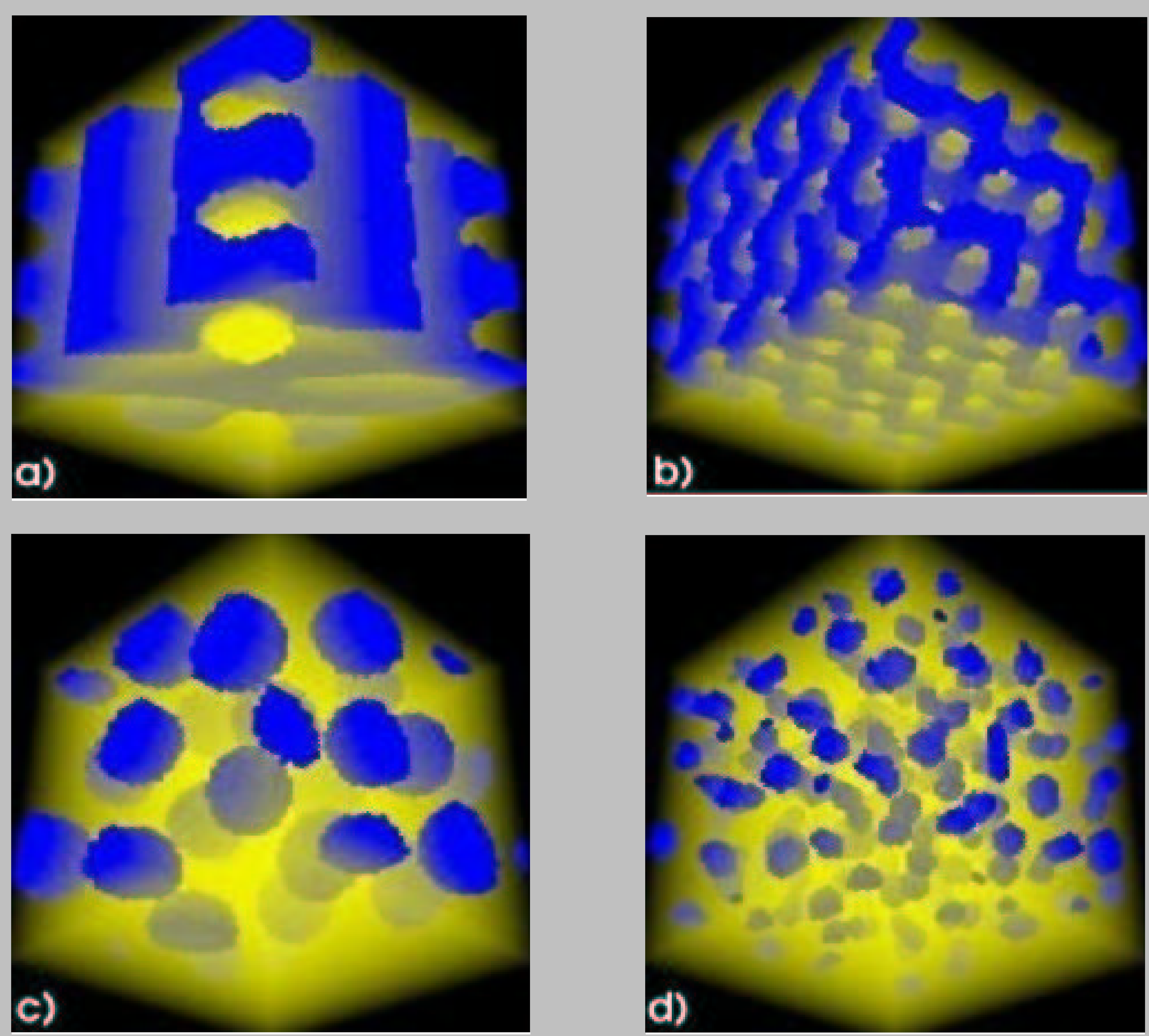
We have carried out extensive numerical studies of Turing systems in both two and three dimensions [2]. The comparison between these two cases is of interest. The morphological development becomes more complex in three dimensions as illustrated in the figures below.

2D domain 100x100 with four different sets of parameters:

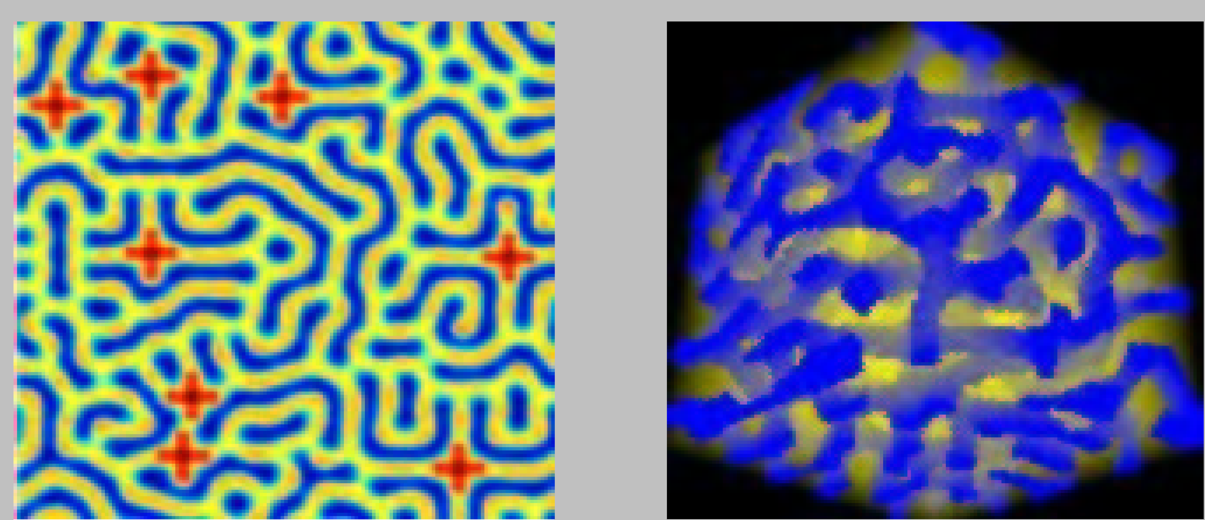


a) b) c) d)

3D domain 50x50x50 with the same four sets of parameters:



We have also studied a Turing system, which generates complex networks by building up connections between sources of chemicals. Based on this observation we are developing a model to explain some features of neural patterning. The figures below show networks generated by a Turing model in 2D and 3D:



References

[1] A.M. Turing: The Chemical Basis of Morphogenesis, *Phil. Trans. R. Soc. Lond.* **B237**, 37−72 (1952).
[2] T. Leppänen, M. Karttunen, K. Kaski, R.A. Barrio, and L. Zhang: A new dimension to Turing patterns, *to be published in Physica D* (2002).